

VERTICES OF THE $D_s^{(*)}D^{(*)}K^*$

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Abstract

Taking into account the contributions of the quark-quark, quark-gluon, and gluon-gluon condensate corrections, the strong form factors and coupling constants of $D_{s0}^*D_0^*K^*$, D_sDK^* , $D_s^*D^*K^*$, $D_{s1}D_1K^*$, $D_sD^*K^*$, and $D_{s0}D_1K^*$ vertices are investigated within the three-point QCD sum rules method, without and with the $SU_f(3)$ symmetry.

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I. INTRODUCTION

In high energy physics, investigation of meson interactions depends on information about the proper functional form of the strong form factors. Among all vertices, the charmed meson ones, which play an important role in understanding the final state interactions, are much more significant. Therefore, researchers have concentrated on computing the strong form factors and coupling constants connected to these vertices. Until now, the vertices involving charmed mesons such as $D^*D^*\rho$ [1], $D^*D\pi$ [2, 3], $DD\rho$ [4], $D^*D\rho$ [5], DDJ/ψ [6], D^*DJ/ψ [7], D^*D_sK , D_s^*DK , D_0D_sK , $D_{s0}DK$ [8], D^*D^*P , D^*DV , DDV [9], $D^*D^*\pi$ [10], D_sD^*K , D_s^*DK [11], $DD\omega$ [12], D_sD_sV , $D_s^*D_s^*V$ [13, 14], and $D_1D^*\pi$, $D_1D_0\pi$, $D_1D_1\pi$ [15] have been studied within the framework of the QCD sum rules.

The effective Lagrangians for the interaction vertices D_sDK^* , $D_sD^*K^*$, and $D_s^*D^*K^*$ are [16]:

$$\begin{aligned}\mathcal{L}_{D_sDK^*} &= ig_{D_sDK^*} K^{*\alpha} (\bar{D}_s \partial_\alpha D - \partial_\alpha \bar{D}_s D), \\ \mathcal{L}_{D_sD^*K^*} &= -g_{D_sD^*K^*} \epsilon^{\alpha\beta\rho\sigma} \partial_\alpha D_\beta^* (\partial_\rho K_\sigma^* \bar{D}_s + D_s \partial_\rho \bar{K}_\sigma^*), \\ \mathcal{L}_{D_s^*D^*K^*} &= ig_{D_s^*D^*K^*} [D_s^{*\mu} (\partial_\mu K^{*\nu} \bar{D}_\nu^* - K^{*\nu} \partial_\mu \bar{D}_\nu^*) + (\partial_\mu D_s^{*\nu} K_\nu^* - D_s^{*\nu} \partial_\mu K_\nu^*) \bar{D}^{*\mu} \\ &\quad + K^{*\mu} (D_s^{*\nu} \partial_\mu \bar{D}_\nu^* - \partial_\mu D_s^{*\nu} \bar{D}_\nu^*)],\end{aligned}\tag{1}$$

where $g_{D_sDK^*}$, $g_{D_sD^*K^*}$, and $g_{D_s^*D^*K^*}$ are the strong form factors. From these Lagrangians, the elements related to the D_sDK^* , $D_sD^*K^*$ and $D_s^*D^*K^*$ vertices can be derived in terms of the strong form factors as:

$$\begin{aligned}\langle D(p) D_s(p') | K^*(q, \varepsilon'') \rangle &= -g_{D_sDK^*} (q^2) \times (p^\mu + p'^\mu) \varepsilon''_\mu, \\ \langle D^*(p, \varepsilon) D_s(p') | K^*(q, \varepsilon'') \rangle &= ig_{D_sD^*K^*} (q^2) \times \epsilon^{\alpha\beta\mu\nu} p'_\alpha q_\beta \varepsilon_\mu(p) \varepsilon''_\nu(q), \\ \langle D^*(p, \varepsilon) D_s^*(p', \varepsilon') | K^*(q, \varepsilon'') \rangle &= ig_{D_s^*D^*K^*} (q^2) \times [(q^\alpha + p'^\alpha) g^{\mu\nu} - (q^\mu + p'^\mu) g^{\nu\alpha} + q^\nu g^{\alpha\mu}] \\ &\quad \times \varepsilon_\alpha(p) \varepsilon'_\mu(p') \varepsilon''_\nu(q),\end{aligned}\tag{2}$$

where $q = p - p'$.

In this work, we decide to calculate the strong form factors and coupling constants associated with the $D_{s0}^*D_0^*K^*$, D_sDK^* , $D_s^*D^*K^*$, $D_{s1}D_1K^*$, $D_sD^*K^*$, and $D_{s0}^*D_1K^*$ vertices in the frame work of the three-point QCD sum rules (3PSR).

The plan of the present paper is as follows: In Section II, the strong form factor calculation of the D_sDK^* vertex is derived in the frame work of the 3PSR; computing the

quark-quark, quark-gluon and gluon-gluon condensate contributions in the Borel transform scheme. In Section III, using necessary changes in the expression obtained for the $g_{D_s DK^*}$, the strong form factors $g_{D_{s0}^* D_0^* K^*}$, $g_{D_s^* D^* K^*}$, $g_{D_{s1} D_1 K^*}$, $g_{D_s D^* K^*}$, and $g_{D_{s0}^* D_1 K^*}$ are presented. The next section depicts our numerical analysis of the strong form factors as well as the coupling constants, without and with the $SU_f(3)$ symmetry.

II. THE STRONG FORM FACTOR OF $D_s DK^*$ VERTEX

To compute the strong form factor of the $D_s DK^*$ vertex via the 3PSR, we start with the correlation function. When K^* meson is off-shell, the correlation function is as follows.

$$\Pi_\mu^{K^*}(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j^{D_s}(x) j_\mu^{K^* \dagger}(0) j^{D \dagger}(y) \} | 0 \rangle. \quad (3)$$

For off-shell charmed meson, the correlation function is:

$$\Pi_\mu^D(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j^{D_s}(x) j^{D \dagger}(0) j_\mu^{K^* \dagger}(y) \} | 0 \rangle, \quad (4)$$

where $j^{D_s} = \bar{c}\gamma_5 s$, $j^D = \bar{c}\gamma_5 u$, and $j_\mu^{K^*} = \bar{u}\gamma_\mu s$ are interpolating currents with the same quantum numbers of D_s , D , and K^* mesons, respectively. Also \mathcal{T} is time ordering product, p and p' are the four momentum of the initial and final mesons, respectively as depicted in Fig. 1.

The correlation functions in Eqs. (3) and (4) are complex functions of which the real part comprises the computations of the theoretical part or QCD and imaginary part comprises the computations of the physical or phenomenological. In the QCD representation, the correlation function is evaluated in quark-gluon language like quark-quark, gluon-gluon condensate, etc using the Wilson operator product expansion (OPE). In the phenomenological part, the representation is in terms of hadronic degrees of freedom which is responsible for the introduction of the form factors, decay constants and masses.

The QCD part of the correlation functions can be calculated by expanding it in terms of the OPE, in the deep Euclidean region, as:

$$\Pi_\mu = C_\mu^{(0)} \mathbf{I} + C_\mu^{(3)} \langle 0 | \bar{\Psi} \Psi | 0 \rangle + C_\mu^{(4)} \langle 0 | G_{\rho\nu}^a G_a^{\rho\nu} | 0 \rangle + C_\mu^{(5)} \langle 0 | \bar{\Psi} \sigma_{\rho\nu} T^a G_a^{\rho\nu} \Psi | 0 \rangle + \dots, \quad (5)$$

where $C_\mu^{(i)}$ are the Wilson coefficients, \mathbf{I} is the unit operator, $\bar{\Psi}$ is the local fermion field operator and $G^{\rho\nu}$ is the gluon strength tensor. The Wilson coefficient $C_\mu^{(0)}$ is contribution

of the perturbative part of the QCD and the other coefficients are contribution of the non-perturbative part. The diagrams corresponding to the perturbative (bare loop), are depicted in Fig. 1.

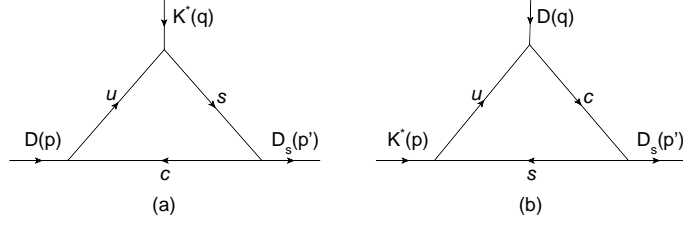


FIG. 1: Perturbative diagrams for off-shell K^* (a) and off-shell D (b).

To compute the portion of the perturbative part using the double dispersion relation for coefficient of the Lorentz structure p_μ appearing in Eq. (2), we obtain:

$$C^{(0)}(p^2, p'^2, q^2) = -\frac{1}{4\pi^2} \int ds \int ds' \frac{\rho(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \quad (6)$$

where ρ is spectral density. Performing the Fourier transformation and using the Cutkosky rules, i.e., $\frac{1}{p^2 - m^2} \rightarrow -2i\pi\delta(p^2 - m^2)$, the spectral densities are calculated for the p_μ structure related to the $D_s D K^*$ vertex.

- For the off-shell K^* (Fig. 1 (a)):

$$\rho_{D_s D K^*}^{K^*} = 6I_0[2m_c m_s - 2m_c^2 + \Delta' + C_1'(2m_c m_s - 2m_c^2 + u)].$$

- For the off-shell D (Fig. 1 (b)):

$$\rho_{D_s D K^*}^D = 6I_0[2m_c m_s - 2m_s^2 + \Delta + C_1(2m_c m_s - 2m_s^2 + 2\Delta + u)].$$

The explicit expressions of the coefficients in the spectral densities are given in Appendix-A.

To compute the contribution of the non-perturbative part of the correlation function for the off-shell K^* meson, six diagrams of dimension 4 are considered. These diagrams named gluon-gluon condensate, shown in Fig. 2. In this case the gluon-gluon diagrams are more important than the other terms in the OPE, since the heavy c quark is a spectator [17]. When D is off-shell, the quark-quark and quark-gluon diagrams of dimension 3 and 5 are more important than the gluon-gluon condensate, since the light s quark is a spectator [17]. Fig. 3 shows these diagrams related to the quark-quark and quark-gluon condensate.

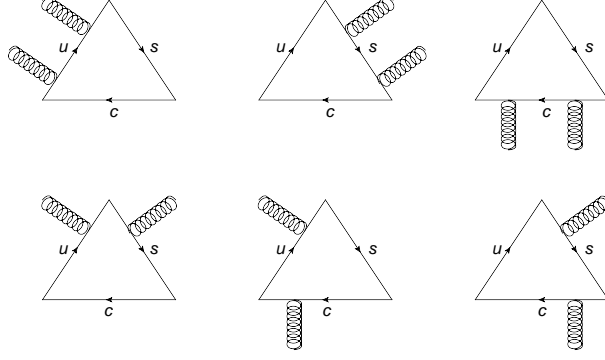


FIG. 2: Non-perturbative diagrams for the off-shell K^* meson.

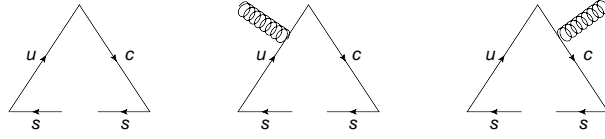


FIG. 3: Non-perturbative diagrams for the off-shell D meson.

After some straightforward calculations and applying the double Borel transformations with respect to the $p^2(p^2 \rightarrow M_1^2)$ and $p'^2(p'^2 \rightarrow M_2^2)$, where M_1^2 and M_2^2 are Borel parameters, the following results are obtained for the non-perturbative contributions corresponding to Fig. 2 and Fig. 3, respectively:

$$C^{(4)} = i \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{D_s D K^*}^{K^*}}{12}, \quad C^{(3)} + C^{(5)} = \langle s\bar{s} \rangle \frac{C_{D_s D K^*}^D}{12}, \quad (7)$$

where the explicit expressions for $C_{D_s D K^*}^{K^*}$ and $C_{D_s D K^*}^D$ are given in appendix-B. It should be noted that to obtain the gluon-gluon condensate contributions, we will follow the same procedure as stated in [18].

In order to calculate the phenomenological part of the correlation functions in Eqs. (3) and (4), three complete sets of intermediate states with the same quantum number should be inserted in these equations. Performing the Fourier transformation, for the phenomenological parts, we have:

$$\begin{aligned} \Pi_\mu^{K^*} &= \frac{\langle 0 | j^{D_s} | D_s(p') \rangle \langle 0 | j^D | D(p) \rangle \langle D_s(p') D(p) | K^*(q, \epsilon) \rangle \langle K^*(q, \epsilon) | j_\mu^{K^*} | 0 \rangle}{(p^2 - m_D^2)(p'^2 - m_{D_s}^2)(q^2 - m_{K^*}^2)} \\ &\quad + \text{higher and continuum states,} \\ \Pi_\mu^D &= \frac{\langle 0 | j^{D_s} | D_s(p') \rangle \langle 0 | j_\mu^{K^*} | K^*(p, \epsilon) \rangle \langle D_s(p') K^*(p, \epsilon) | D(q) \rangle \langle D(q) | j^D | 0 \rangle}{(p^2 - m_{K^*}^2)(p'^2 - m_{D_s}^2)(q^2 - m_D^2)} \\ &\quad + \text{higher and continuum states.} \end{aligned} \quad (8)$$

The matrix elements $\langle 0 | j_\mu^{K^*} | K^*(q, \varepsilon) \rangle$, and $\langle 0 | j^{D(s)} | D(s)(p) \rangle$ are defined as:

$$\begin{aligned}\langle 0 | j_\mu^{K^*} | K^*(q, \varepsilon) \rangle &= m_{K^*} f_{K^*} \varepsilon_\mu(q), \\ \langle 0 | j^{D(s)} | D(s)(p) \rangle &= \frac{m_{D(s)}^2 f_{D(s)}}{m_c + m_{u(s)}},\end{aligned}\quad (9)$$

where m_{K^*} , $m_{D(s)}$, f_{K^*} , and $f_{D(s)}$ are the masses and decay constants of mesons K^* and $D(s)$, respectively. ε_μ is the polarization vector of the vector meson K^* .

Inserting Eqs. (2) and (9) in Eq. (8) and after some calculations, we obtain $\Pi_\mu^{K^*}$ and Π_μ^D in terms of strong form factors $g_{D_s DK^*}^{K^*}$ and $g_{D_s DK^*}^D$ as:

$$\begin{aligned}\Pi_\mu^{K^*} &= -g_{D_s DK^*}^{K^*}(q^2) \frac{m_{K^*} m_D^2 m_{D_s}^2 f_{K^*} f_D f_{D_s}}{m_c(m_c + m_s)(p^2 - m_D^2)(p'^2 - m_{D_s}^2)(q^2 - m_{K^*}^2)} p_\mu + \dots \\ \Pi_\mu^D &= -g_{D_s DK^*}^D(q^2) \frac{m_{D_s}^2 m_D^2 f_{K^*} f_D f_{D_s} (m_{D_s}^2 + m_{K^*}^2 - q^2)}{m_c(m_c + m_s) m_{K^*} (p^2 - m_{K^*}^2)(p'^2 - m_{D_s}^2)(q^2 - m_D^2)} p_\mu + \dots\end{aligned}\quad (10)$$

The strong form factors are calculated by equating two representations of the correlation function and applying the Borel transformations with respect to the $p^2(p^2 \rightarrow M_1^2)$ and $p'^2(p'^2 \rightarrow M_2^2)$ on the phenomenological as well as the perturbative and nonperturbative parts of the correlation function in order to suppress the contributions of the higher states and continuum. The equations for the strong form factors are obtained as follows:

$$\begin{aligned}g_{D_s DK^*}^{K^*}(q^2) &= \Lambda_{D_s DK^*}^{K^*} \left\{ -\frac{1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0^{D_s}} ds' \int_{s_1}^{s_0^D} ds \rho_{DD_s K^*}^{K^*}(s, s', q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} \right. \\ &\quad \left. - i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{D_s DK^*}^{K^*}}{12} \right\}, \\ g_{D_s DK^*}^D(q^2) &= \Lambda_{D_s DK^*}^D \left\{ -\frac{1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0^{D_s}} ds' \int_{s_2}^{s_0^{K^*}} ds \rho_{DD_s K^*}^D(s, s', q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} \right. \\ &\quad \left. + M_1^2 M_2^2 \langle s\bar{s} \rangle \frac{C_{D_s DK^*}^D}{12} \right\},\end{aligned}\quad (11)$$

where $s_0^{K^*}$ and $s_0^{D(D_s)}$ are the continuum thresholds in K^* and $D(D_s)$ mesons, respectively. s_1 and s_2 are the lower limits of the integrals over s as

$$s_1 = \frac{m_c^2(m_c^2 - s' + q^2)}{m_c^2 - s'}, \quad s_2 = \frac{m_s^2(m_s^2 - s' + q^2)}{m_s^2 - s'}.$$

Also $\Lambda_{D_s DK^*}^{K^*}$ and $\Lambda_{D_s DK^*}^D$ are defined as:

$$\begin{aligned}\Lambda_{D_s DK^*}^{K^*} &= -\frac{m_c(m_c + m_s)(q^2 - m_{K^*}^2)}{m_{K^*} m_D^2 m_{D_s}^2 f_{K^*} f_D f_{D_s}} e^{\frac{m_D^2}{M_1^2}} e^{\frac{m_{D_s}^2}{M_2^2}}, \\ \Lambda_{D_s DK^*}^D &= -\frac{m_c(m_c + m_s) m_{K^*} (q^2 - m_D^2)}{m_D^2 m_{D_s}^2 f_{K^*} f_D f_{D_s} (m_{D_s}^2 + m_{K^*}^2 - q^2)} e^{\frac{m_{K^*}^2}{M_1^2}} e^{\frac{m_{D_s}^2}{M_2^2}}.\end{aligned}\quad (12)$$

III. OTHER VERTICES AND STRONG FORM FACTORS

Following the previous steps in section II, phrases similar to Eq. (11) can be obtained for the strong form factors of the $D_{s0}^* D_0^* K^*$, $D_s^* D^* K^*$, $D_{s1} D_1 K^*$, $D_s D^* K^*$, and $D_{s0}^* D_1 K^*$ vertices via the 3PSR. For this purpose, the appropriate terms of Λ , the spectral density, and quark-gluon condensate should be replaced in Eq. (11). proper expressions for Λ , the spectral density, and quark-gluon condensate, related to the strong form factors $g_{D_{s0}^* D_0^* K^*}$, $g_{D_s^* D^* K^*}$, $g_{D_{s1} D_1 K^*}$, $g_{D_s D^* K^*}$, and $g_{D_{s0}^* D_1 K^*}$, have been listed in the Tables I-II and Appendix-B, respectively.

TABLE I: Expressions for the coefficient Λ related to the strong form factors $g_{D_{s0}^* D_0^* K^*}$, $g_{D_s^* D^* K^*}$, $g_{D_{s1} D_1 K^*}$, $g_{D_s D^* K^*}$, and $g_{D_{s0}^* D_1 K^*}$.

Coefficient(Λ)	Expression
$\Lambda_{D_{s0}^* D_0^* K^*}^{D_0^*}$	$-\frac{m_{K^*}(q^2-m_{D_0^*}^2)}{m_{D_{s0}^*} m_{D_0^*} f_{K^*} f_{D_{s0}^*} f_{D_0^*} (m_{D_{s0}^*}^2 + m_{K^*}^2 - q^2)} e^{\frac{m_{K^*}^2}{M_1^2}} e^{\frac{m_{D_{s0}^*}^2}{M_2^2}}$
$\Lambda_{D_{s0}^* D_0^* K^*}^{K^*}$	$-\frac{(q^2-m_{K^*}^2)}{m_{K^*} m_{D_{s0}^*} m_{D_0^*} f_{K^*} f_{D_{s0}^*} f_{D_0^*}} e^{\frac{m_{D_0^*}^2}{M_1^2}} e^{\frac{m_{D_{s0}^*}^2}{M_2^2}}$
$\Lambda_{D_s^* D^* K^*}^{D_s^*}$	$-\frac{2m_{D_s^*}(q^2-m_{D^*}^2)}{m_{K^*} f_{K^*} f_{D^*} f_{D_s^*} (3m_{D^*}^2 + m_{K^*}^2 - q^2)} e^{\frac{m_{K^*}^2}{M_1^2}} e^{\frac{m_{D_s^*}^2}{M_2^2}}$
$\Lambda_{D_s^* D^* K^*}^{K^*}$	$-\frac{2m_{D_s^*}(q^2-m_{K^*}^2)}{m_{K^*} f_{K^*} f_{D^*} f_{D_s^*} (3m_{D_s^*}^2 + m_{D^*}^2 - q^2)} e^{\frac{m_{D_s^*}^2}{M_1^2}} e^{\frac{m_{D^*}^2}{M_2^2}}$
$\Lambda_{D_{s1}^* D_1 K^*}^{D_1}$	$-\frac{2m_{D_{s1}^*}(q^2-m_{D_1}^2)}{m_{K^*} f_{K^*} f_{D_1} f_{D_{s1}^*} (3m_{D_{s1}^*}^2 + m_{K^*}^2 - q^2)} e^{\frac{m_{K^*}^2}{M_1^2}} e^{\frac{m_{D_{s1}^*}^2}{M_2^2}}$
$\Lambda_{D_{s1}^* D_1 K^*}^{K^*}$	$-\frac{2m_{D_{s1}^*}(q^2-m_{K^*}^2)}{m_{K^*} f_{K^*} f_{D_1} f_{D_{s1}^*} (3m_{D_{s1}^*}^2 + m_{D_1}^2 - q^2)} e^{\frac{m_{D_1}^2}{M_1^2}} e^{\frac{m_{D_{s1}^*}^2}{M_2^2}}$
$\Lambda_{D_s D^* K^*}^{D_s^*}$	$-\frac{(m_c+m_s)(q^2-m_{D^*}^2)}{m_{K^*} m_{D^*} m_{D_s^*}^2 f_{K^*} f_{D^*} f_{D_s}} e^{\frac{m_{K^*}^2}{M_1^2}} e^{\frac{m_{D_s^*}^2}{M_2^2}}$
$\Lambda_{D_s D^* K^*}^{K^*}$	$-\frac{(m_c+m_s)(q^2-m_{K^*}^2)}{m_{K^*} m_{D^*} m_{D_s^*}^2 f_{K^*} f_{D^*} f_{D_s}} e^{\frac{m_{D^*}^2}{M_1^2}} e^{\frac{m_{D_s^*}^2}{M_2^2}}$
$\Lambda_{D_{s0}^* D_1 K^*}^{D_1}$	$-\frac{(q^2-m_{D_1}^2)}{m_{K^*} m_{D_1} m_{D_{s0}^*} f_{K^*} f_{D_1} f_{D_{s0}^*}} e^{\frac{m_{K^*}^2}{M_1^2}} e^{\frac{m_{D_{s0}^*}^2}{M_2^2}}$
$\Lambda_{D_{s0}^* D_1 K^*}^{K^*}$	$-\frac{(q^2-m_{K^*}^2)}{m_{K^*} m_{D_1} m_{D_{s0}^*} f_{K^*} f_{D_1} f_{D_{s0}^*}} e^{\frac{m_{D_1}^2}{M_1^2}} e^{\frac{m_{D_{s0}^*}^2}{M_2^2}}$

TABLE II: Spectral density expressions connected to the strong form factors $g_{D_{s0}^* D_0^* K^*}$, $g_{D_s^* D^* K^*}$, $g_{D_{s1} D_1 K^*}$, $g_{D_s D^* K^*}$, and $g_{D_{s0}^* D_1 K^*}$.

Spectral Density (ρ)	Expression
$\rho_{D_{s0}^* D_0^* K^*}^{D_0^*}$	$6I_0[2m_c m_s + 2m_s^2 - \Delta + C_1(2m_c m_s + 2m_s^2 - 2\Delta + u)]$
$\rho_{D_{s0}^* D_0^* K^*}^{K^*}$	$6I_0[2m_c m_s + 2m_c^2 - \Delta' + C'_1(2m_c m_s + 2m_c^2 - u)]$
$\rho_{D_s^* D^* K^*}^{D_s^*}$	$3I_0[3m_s^2 - 2m_c m_s - s - \Delta + 4A + (C_1 - C_2)(2u + 2m_c m_s - 2s) - 8(E_1 - E_2)]$
$\rho_{D_s^* D^* K^*}^{K^*}$	$3I_0[3m_c^2 - 2m_c m_s - s - \Delta' + 4A' + 2(C'_1 - C'_2)(u + 2m_c m_s - 2s) - 8(E'_1 - E'_2)]$
$\rho_{D_{s1}^* D_1 K^*}^{D_1}$	$3I_0[3m_s^2 + 2m_c m_s - s - \Delta + 4A + (C_1 - C_2)(2u - 2m_c m_s - 2s) - 8(E_1 - E_2)]$
$\rho_{D_{s1}^* D_1 K^*}^{K^*}$	$3I_0[3m_c^2 + 2m_c m_s - s - \Delta' + 4A' + 2(C'_1 - C'_2)(u - 2m_c m_s - 2s) - 8(E'_1 - E'_2)]$
$\rho_{D_s^* D^* K^*}^{D_s^*}$	$12I_0[C_1 m_s + C_2(m_s - m_c) + m_s]$
$\rho_{D_s^* D^* K^*}^{K^*}$	$-12I_0[C'_1 m_c + C'_2(m_c - m_s) + m_c]$
$\rho_{D_{s0}^* D_1 K^*}^{D_1}$	$12I_0[C_1 m_s + C_2(m_s + m_c) + m_s]$
$\rho_{D_{s0}^* D_1 K^*}^{K^*}$	$-12I_0[C'_1 m_c + C'_2(m_c + m_s) + m_c]$

IV. NUMERICAL ANALYSIS

In this section, the strong form factors, and coupling constants for the $D_{s0}^* D_0^* K^*$, $D_s D K^*$, $D_s^* D^* K^*$, $D_{s1} D_1 K^*$, $D_s D^* K^*$, and $D_{s0}^* D_1 K^*$ vertices are considered. For this aim, the values of quark and meson masses are chosen as: $m_s = 0.14 \pm 0.01$ GeV, $m_{K^*} = 0.89$ GeV, $m_{D_{s0}^*} = 2.32$ GeV, $m_{D_s} = 1.97$ GeV, $m_{D_s^*} = 2.11$ GeV, $m_{D_{s1}} = 2.46$ GeV, $m_{D_0^*} = 2.40$ GeV, $m_D = 1.87$ GeV, $m_{D^*} = 2.01$ GeV and $m_{D_1} = 2.42$ GeV [19]. Also the leptonic decay constants for these vertexes are presented in Table III.

TABLE III: The leptonic decay constants in MeV.

f_{K^*} [19]	$f_{D_{s0}^*}$ [20]	$f_{D_0^*}$ [21]	f_{D_s} [22]	f_D [9]	$f_{D_s^*}$ [20]	f_{D^*} [23]	$f_{D_{s1}}$ [24]	f_{D_1} [25]
220 ± 5	230 ± 20	334 ± 9	294 ± 27	223 ± 17	266 ± 32	340 ± 12	225 ± 20	219 ± 11

There are four auxiliary parameters containing the Borel mass parameters M_1 and M_2 and continuum thresholds $s_0^{K^*}$ and $s_0^{D(D_s)}$ in Eq. (11). The strong form factors and coupling constants are the physical quantities and should be independent of them. However the continuum thresholds are not completely arbitrary; these are related to the energy of the first excited state. The values of the continuum thresholds are taken to be $s_0^{K^*} = (m_{K^*} + \delta)^2$ and $s_0^{D(D_s)} = (m_{D(D_s)} + \delta)^2$. We use $0.4 \text{ GeV} \leq \delta \leq 0.6 \text{ GeV}$ in $Q^2 = 1 \text{ GeV}^2$, where $Q^2 = -q^2$ [1–3].

Our results should be almost insensitive to the intervals of the Borel parameters. On the other hand, the intervals of the Borel mass parameters must suppress the higher states, continuum and contributions of the highest-order operators. In other words, the sum rules for the strong form factors must converge. In this work, the following relations between the Borel masses M_1 and M_2 are used.

- For the off-shell K^* meson:

$$\frac{M_1^2}{M_2^2} = \frac{m_D^2}{m_{D_s}^2}. \quad (13)$$

- For the off-shell D meson:

$$\frac{M_1^2}{M_2^2} = \frac{m_{K^*}^2}{m_{D_s}^2 - m_c^2}. \quad (14)$$

So, only one independent Borel mass parameter, M is obtained according to these relations between the M_1 and M_2 . We found a good stability of the sum rule in the interval

$10 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2$ for all vertices. For instance, the dependence of the strong form factor, $g_{D_s DK^*}$ on Borel mass parameter, M^2 in $Q^2 = 1$ is shown in Fig. 4.

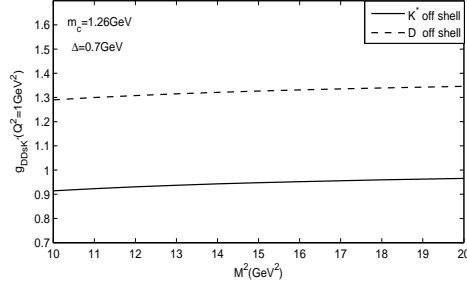


FIG. 4: The strong form factors $g_{D_s DK^*}$ as functions of the Borel mass parameter M^2 .

To extend the Q^2 dependence of the strong form factors to the full physical region, where the sum rule results are not valid, we find that the sum rules predictions for the form factors in Eq. (11) are well fitted to the following function:

$$g(Q^2) = A e^{-Q^2/B}. \quad (15)$$

The values of the parameters A and B are given for two sets in Table IV. set I: $m_c = 1.26 \text{ GeV}$ and set II: $m_c = 1.47 \text{ GeV}$.

The dependence of the strong form factors $g_{D_s DK^*}^D(Q^2)$ and $g_{D_s DK^*}^{K^*}(Q^2)$ in Q^2 are shown in Fig. 5. In this figure, the small circles and boxes correspond to the form factors via the 3PSR calculation. As it is seen, the form factors and their fit functions coincide together, well.

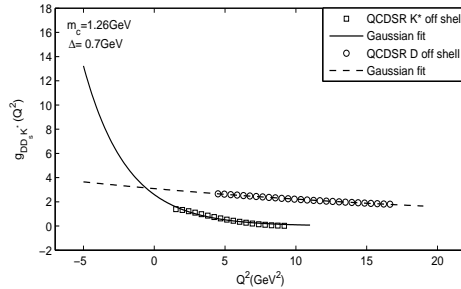


FIG. 5: The strong form factors $g_{D_s DK^*}^D$ and $g_{D_s DK^*}^{K^*}$ on Q^2 .

The value of the strong form factors at $Q^2 = -m_m^2$, where m_m is the mass of the off-shell meson, is defined as coupling constant. Coupling constant results of the vertices,

TABLE IV: Parameters appearing in the fit functions for the $D_{s0}^*D_0^*K^*$, D_sDK^* , $D_s^*D^*K^*$, $D_{s1}D_1K^*$, $D_sD^*K^*$, and $D_{s0}D_1K^*$, vertices for various m_c and δ , where $\delta_1 = 0.4$ GeV, $\delta_2 = 0.7$ GeV and $\delta_3 = 1.0$ GeV.

	set I				set II			
Form factor	$A(\delta_1)$	$B(\delta_1)$	$A(\delta_2)$	$B(\delta_2)$	$A(\delta_3)$	$B(\delta_3)$	$A(\delta_2)$	$B(\delta_2)$
$g_{D_{s0}^*D_0^*K^*}^{K^*}(Q^2)$	5.32	3.24	6.03	4.61	6.96	7.57	4.46	2.21
$g_{D_{s0}^*D_0^*K^*}^{D_0}(Q^2)$	5.33	26.60	5.78	27.16	6.56	28.06	5.22	26.13
$g_{D_s^*DK^*}^{K^*}(Q^2)$	2.22	2.77	2.58	3.07	3.19	4.20	2.21	1.52
$g_{D_s^*DK^*}^D(Q^2)$	2.71	30.28	3.09	30.21	3.49	30.26	3.22	33.93
$g_{D_s^*D^*K^*}^{K^*}(Q^2)$	4.36	191.56	4.91	199.52	5.67	212.43	4.63	438.03
$g_{D_s^*D^*K^*}^{D^*}(Q^2)$	3.58	52.74	4.42	49.58	5.06	47.41	4.20	53.70
$g_{D_{s1}D_1K^*}^{K^*}(Q^2)$	3.78	18.13	4.26	26.95	4.64	40.86	4.52	8.49
$g_{D_{s1}D_1K^*}^{D_1}(Q^2)$	2.76	18.44	3.18	21.05	3.89	26.76	3.31	16.26
$g_{D_sD^*K^*}^{K^*}(Q^2)$	3.98	13.20	4.15	7.34	4.39	6.07	4.26	5.32
$g_{D_sD^*K^*}^{D^*}(Q^2)$	3.98	39.11	4.26	34.29	4.56	28.46	4.42	30.64
$g_{D_{s0}D_1K^*}^{K^*}(Q^2)$	7.32	14.57	8.31	32.56	9.09	56.18	8.58	15.17
$g_{D_{s0}D_1K^*}^{D_1}(Q^2)$	5.33	17.06	5.63	15.60	5.75	12.96	6.35	15.58

$D_{s0}^*D_0^*K^*$, D_sDK^* , $D_s^*D^*K^*$, $D_{s1}D_1K^*$, $D_sD^*K^*$, and $D_{s0}D_1K^*$ are presented in Table V. The errors are estimated by the variation of the Borel parameter, the variation of the continuum thresholds, the leptonic decay constants and uncertainties in the values of the other input parameters. It should be noted that the main uncertainty comes from the continuum thresholds and the decay constants.

Table VI shows a comparison between our results with the values predicted by the light-cone sum rules (LCSR) method. The results of Ref. [26] have been rescaled according to the strong form factor definitions in Eq. (2). It should be reminded that the average value of two coupling constants, $g_{D_sDK^*}^D$ and $g_{D_sDK^*}^{K^*}$ in set I in Table V, are presented in Table VI. As seen, our values are in the reasonable agreement with those of the LCSR.

In order to investigate the strong coupling constant values via the $SU_f(3)$ symmetry, the mass of the s quark are ignored in all equations. In view of the $SU_f(3)$ symmetry, the

TABLE V: The coupling constant of the vertices $D_{s0}^*D_0^*K^*$, D_sDK^* , $D_s^*D^*K^*$, $D_{s1}D_1K^*$, $D_sD^*K^*$, and $D_{s0}D_1K^*$, in GeV^{-1} for various m_c .

g	set I		set II	
	off-shell charmed	off-shell K^*	off-shell charmed	off-shell K^*
$g_{D_{s0}^*D_0^*K^*}$	7.09 ± 0.91	7.16 ± 0.71	6.46 ± 0.88	6.39 ± 0.81
$g_{D_sDK^*}$	3.47 ± 0.45	3.37 ± 0.48	3.57 ± 0.47	3.72 ± 0.42
$g_{D_s^*D^*K^*}$	4.79 ± 0.72	4.93 ± 0.74	4.53 ± 0.66	4.63 ± 0.76
$g_{D_{s1}D_1K^*}$	4.29 ± 1.34	4.42 ± 0.90	4.05 ± 1.13	3.88 ± 1.06
$g_{D_sD^*K^*}$	4.80 ± 0.45	4.62 ± 0.38	5.04 ± 0.37	4.95 ± 0.41
$g_{D_{s0}D_1K^*}$	8.19 ± 0.81	8.51 ± 0.78	9.25 ± 0.76	9.04 ± 0.81

TABLE VI: Values of the strong coupling constant using the 3PSR (ours) and LCSR approaches, in GeV^{-1} .

g	Ours	LCSR [26]
$g_{D_sDK^*}$	3.42 ± 0.44	3.22 ± 0.66
$g_{D_sD^*K^*}$	4.71 ± 0.39	4.04 ± 0.80

values of the parameters A and B for the $g_{D_{s0}^*D_0^*K^*}$, $g_{D_sDK^*}$, $g_{D_s^*D^*K^*}$, $g_{D_{s1}D_1K^*}$, $g_{D_sD^*K^*}$, and $g_{D_{s0}D_1K^*}$ vertices in $m_c = 1.26 \text{ GeV}$ and $\delta = 0.7 \text{ GeV}$ are given in Table VII.

Also considering the $SU_f(3)$ symmetry, we obtain the values of the coupling constants in $m_c = 1.26 \text{ GeV}$ shown in Table VIII.

It is possible to compare the coupling constant values of $g_{D_sDK^*}$, $g_{D_sD^*K^*}$ and $g_{D_s^*D^*K^*}$ with $g_{DD\rho}$, $g_{DD^*\rho}$ and $g_{D^*D^*\rho}$ respectively, in the $SU_f(3)$ symmetry consideration. Table IX shows that results are reasonably consistent to each other.

In summary, taking into account the contributions of the quark-quark, quark-gluon and gluon-gluon condensate corrections, the strong form factors $g_{D_{s0}^*D_0^*K^*}$, $g_{D_sDK^*}$, $g_{D_s^*D^*K^*}$, $g_{D_{s1}D_1K^*}$, $g_{D_sD^*K^*}$, and $g_{D_{s0}D_1K^*}$ were estimated within the 3PSR without and with the $SU_f(3)$ symmetry. For instance, the dependence of the strong form factors $g_{D_sDK^*}$ on the transferred momentum square Q^2 were plotted. Also the coupling constants of these vertices were evaluated.

TABLE VII: Parameters appearing in the fit functions for the $g_{D_{s0}^*D_0^*K^*}$, $g_{D_sDK^*}$, $g_{D_s^*D^*K^*}$, $g_{D_{s1}D_1K^*}$, $g_{D_sD^*K^*}$, and $g_{D_{s0}D_1K^*}$ form factors in $SU_f(3)$ symmetry with $m_c = 1.26$ GeV and $\delta = 0.7$ GeV.

Form factor	A	B	Form factor	A	B
$g_{D_{s0}^*D_0^*K^*}(Q^2)$	6.96	7.57	$g_{D_{s1}^*D_1K^*}(Q^2)$	4.69	26.55
$g_{D_{s0}^*D_0^*K^*}^{D_0^*}(Q^2)$	6.16	27.73	$g_{D_{s1}^*D_1K^*}^{D_1}(Q^2)$	3.62	22.20
$g_{D_sDK^*}^{K^*}(Q^2)$	2.06	2.08	$g_{D_sD^*K^*}^{K^*}(Q^2)$	3.43	7.88
$g_{D_sDK^*}^D(Q^2)$	2.62	33.57	$g_{D_sD^*K^*}^{D^*}(Q^2)$	3.08	16.78
$g_{D_s^*D^*K^*}^{K^*}(Q^2)$	5.16	220.80	$g_{D_{s0}^*D_1K^*}^{K^*}(Q^2)$	9.02	29.52
$g_{D_s^*D^*K^*}^{D^*}(Q^2)$	4.81	49.58	$g_{D_{s0}^*D_1K^*}^{D_1}(Q^2)$	7.06	19.32

TABLE VIII: The coupling constant of the vertices $D_{s0}^*D_0^*K^*$, D_sDK^* , $D_s^*D^*K^*$, $D_{s1}D_1K^*$, $D_sD^*K^*$, and $D_{s0}D_1K^*$, in $SU_f(3)$ symmetry, in GeV^{-1} .

g	off-shell charmed	off-shell K^*	g	off-shell charmed	off-shell K^*
$g_{D_{s0}^*D_0^*K^*}$	7.53 ± 0.82	7.73 ± 0.63	$g_{D_{s1}D_1K^*}$	4.71 ± 0.32	4.83 ± 0.52
$g_{D_sDK^*}$	2.91 ± 0.37	3.02 ± 0.45	$g_{D_sD^*K^*}$	3.91 ± 0.32	3.79 ± 0.41
$g_{D_s^*D^*K^*}$	5.22 ± 0.71	5.18 ± 0.66	$g_{D_{s0}^*D_1K^*}$	9.56 ± 1.12	9.27 ± 0.92

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TABLE IX: Values of the coupling constant using the LCSR, and 3PSR.

g	Ours	3PSR [4–6]	LCSR [26]
$g_{D_s DK^*}$	2.95 ± 0.44	3.42 ± 0.44	2.62 ± 0.66
$g_{D_s^* D^* K^*}$	5.20 ± 0.70	6.60 ± 0.30	—
$g_{D_s D^* K^*}$	3.81 ± 0.39	4.11 ± 0.44	3.56 ± 0.60

Appendix–A

In this appendix, the explicit expressions of the coefficients in the spectral densities are given as:

$$\begin{aligned}
I_0(s, s', q^2) &= \frac{1}{4\lambda^{\frac{1}{2}}(s, s', q^2)}, \\
\lambda(a, b, c) &= a^2 + b^2 + c^2 - 2ac - 2bc - 2ac, \\
\Delta &= s' + m_s^2 - m_c^2, \\
\Delta' &= s' + m_c^2 - m_s^2, \\
\Delta'' &= s + m_s^2, \\
u &= s + s' - q^2, \\
C_1 &= \frac{1}{\lambda(s, s', q^2)} [2s'\Delta'' - u\Delta], \\
C_2 &= \frac{1}{\lambda(s, s', q^2)} [2s\Delta - u\Delta''], \\
A &= -\frac{1}{2\lambda(s, s', q^2)} [4ss'm_s^2 - s\Delta^2 - s'\Delta''^2 - m_s^2u^2 + u\Delta\Delta''], \\
E_1 &= \frac{1}{2\lambda^2(s, s', q^2)} [8ss'^2m_s^2\Delta'' - 2s'm_s^2u^2\Delta'' - 4ss'm_s^2u\Delta + m_s^2u^3\Delta - 2s'^2\Delta''^3 \\
&\quad + 3s'u\Delta\Delta''^2 - 2ss'\Delta^2\Delta'' - u^2\Delta^2\Delta'' + su\Delta^3], \\
E_2 &= \frac{1}{2\lambda^2(s, s', q^2)} [8s^2s'm_s^2\Delta - 2sm_s^2u^2\Delta'' - 4ss'm_s^2u\Delta'' + m_s^2u^3\Delta'' - 2s^2\Delta^3 \\
&\quad + 3su\Delta^2\Delta'' - 2ss'\Delta\Delta''^2 - u^2\Delta\Delta''^2 + s'u\Delta''^3],
\end{aligned}$$

also $A' = A|_{m_c \leftrightarrow m_s}$, $C_1' = C_1|_{m_c \leftrightarrow m_s}$, $C_2' = C_2|_{m_c \leftrightarrow m_s}$, $E_1' = E_1|_{m_c \leftrightarrow m_s}$, and $E_2' = E_2|_{m_c \leftrightarrow m_s}$.

Appendix–B

In this appendix, the explicit expressions of the coefficients of the quark and gluon condensate contributions of the strong form factors in the Borel transform scheme for all the vertices are presented.

$$C_{D_{s0}^* D_0^* K^*}^{D_0^*} = \left(3 \frac{m_s q^2}{M_1^2} - 3 \frac{m_s m_c^2}{M_1^2} + \frac{m_0^2 m_c}{M_1^2} - 3 \frac{m_s m_c^2}{M_2^2} - 6 \frac{m_c m_s^2}{M_2^2} - 3 \frac{m_c q^2 m_s^2}{M_1^2 M_2^2} + 3 \frac{m_c^3 m_s^2}{M_1^2 M_2^2} \right. \\ \left. - 2 \frac{m_0^2 m_c^3}{M_1^2 M_2^2} + 2 \frac{m_0^2 m_c q^2}{M_1^2 M_2^2} + 3 \frac{m_c^3 m_s^2}{M_2^4} - \frac{3}{2} \frac{m_0^2 m_c^3}{M_2^4} \right) \times e^{-\frac{m_c^2}{M_2^2}},$$

$$C_{D_s D K^*}^D = \left(3 \frac{m_s m_c^2}{M_1^2} - 3 \frac{m_s q^2}{M_1^2} + 3 \frac{m_c m_s^2}{M_1^2} - \frac{5}{2} \frac{m_0^2 m_c}{M_1^2} + 3 \frac{m_s m_c^2}{M_2^2} - 3 \frac{m_0^2 m_c}{M_2^2} - \frac{1}{2} \frac{m_0^2 m_c^3}{M_1^2 M_2^2} \right. \\ \left. + \frac{1}{2} \frac{m_0^2 m_c q^2}{M_1^2 M_2^2} + 3 \frac{m_c^3 m_s^2}{M_2^4} - \frac{3}{2} \frac{m_0^2 m_c^3}{M_2^4} \right) \times e^{-\frac{m_c^2}{M_2^2}},$$

$$C_{D_s^* D^* K^*}^{D^*} = \left(-6 \frac{m_s m_c^2}{M_2^2} - 2 \frac{m_0^2 m_c}{M_2^2} + 6 \frac{m_c m_s^2}{M_2^2} + 3 \frac{m_s q^2}{M_2^2} + 3 \frac{m_c q^2 m_s^2}{M_1^2 M_2^2} - \frac{m_0^2 m_c q^2}{M_1^2 M_2^2} - 3 \frac{m_c^3 m_s^2}{M_1^2 M_2^2} \right. \\ \left. + \frac{m_0^2 m_c^3}{M_1^2 M_2^2} - 3 \frac{m_c^3 m_s^2}{M_2^4} + \frac{3}{2} \frac{m_0^2 m_c^3}{M_2^4} + 3 m_s \right) \times e^{-\frac{m_c^2}{M_2^2}},$$

$$C_{D_{s1}^* D_1 K^*}^{D_1} = \left(2 \frac{m_0^2 m_c}{M_2^2} - 6 \frac{m_s m_c^2}{M_2^2} + 3 \frac{m_s q^2}{M_2^2} - 6 \frac{m_c m_s^2}{M_2^2} - \frac{m_0^2 m_c^3}{M_1^2 M_2^2} - 3 \frac{m_c q^2 m_s^2}{M_1^2 M_2^2} + \frac{m_0^2 m_c q^2}{M_1^2 M_2^2} \right. \\ \left. + 3 \frac{m_c^3 m_s^2}{M_1^2 M_2^2} + 3 \frac{m_c^3 m_s^2}{M_2^4} - \frac{3}{2} \frac{m_0^2 m_c^3}{M_2^4} + 3 m_s \right) \times e^{-\frac{m_c^2}{M_2^2}},$$

$$C_{D_s D^* K^*}^{D^*} = \left(6 \frac{m_s m_c}{M_2^2} - 6 \frac{q^2 m_s^2}{M_1^2 M_2^2} + 2 \frac{m_0^2 q^2}{M_1^2 M_2^2} + 6 \frac{m_c^2 m_s^2}{M_1^2 M_2^2} - 2 \frac{m_0^2 m_c^2}{M_1^2 M_2^2} + 6 \frac{m_c^2 m_s^2}{M_2^4} \right. \\ \left. - 3 \frac{m_0^2 m_c^2}{M_2^4} \right) \times e^{-\frac{m_c^2}{M_2^2}},$$

$$C_{D_{s0}^* D_1 K^*}^{D_1} = \left(-6 \frac{m_s m_c}{M_2^2} - 6 \frac{q^2 m_s^2}{M_1^2 M_2^2} + 2 \frac{m_0^2 q^2}{M_1^2 M_2^2} + 6 \frac{m_c^2 m_s^2}{M_1^2 M_2^2} - 2 \frac{m_0^2 m_c^2}{M_1^2 M_2^2} + 6 \frac{m_c^2 m_s^2}{M_2^4} \right. \\ \left. - 3 \frac{m_0^2 m_c^2}{M_2^4} \right) \times e^{-\frac{m_c^2}{M_2^2}},$$

$$C_{D_{s0}^* D_0^* K^*}^{K^*} = \hat{I}_2(3, 2, 2) m_c^6 + \hat{I}_0(3, 2, 2) m_c^6 - \hat{I}_1(3, 2, 2) m_c^6 + \hat{I}_2(3, 2, 2) m_c^5 m_s$$

$$\begin{aligned}
& +2\hat{I}_0(3, 2, 2)m_c^5m_s - \hat{I}_1(3, 2, 2)m_c^5m_s - \hat{I}_2(3, 2, 2)m_c^4m_s^2 + \hat{I}_1(3, 2, 2)m_c^4m_s^2 \\
& +\hat{I}_1(3, 2, 2)m_c^3m_s^3 - \hat{I}_2(3, 2, 2)m_c^3m_s^3 - \hat{I}_0(3, 2, 2)m_c^2m_s^4 + 3\hat{I}_0(2, 2, 2)m_c^4 \\
& +\hat{I}_2(3, 2, 1)m_c^4 + \hat{I}_0^{[1,0]}(3, 2, 2)m_c^4 - \hat{I}_1(3, 2, 1)m_c^4 + 6\hat{I}_0(4, 1, 1)m_c^3m_s \\
& -\hat{I}_1(3, 1, 2)m_c^3m_s + 2\hat{I}_2(2, 2, 2)m_c^3m_s + \hat{I}_2(3, 1, 2)m_c^3m_s + 2\hat{I}_0^{[1,0]}(3, 2, 2)m_c^3m_s \\
& +2\hat{I}_0(3, 2, 1)m_c^3m_s - 2\hat{I}_1(2, 2, 2)m_c^3m_s + 8\hat{I}_0(3, 1, 2)m_c^2m_s^2 - \hat{I}_1^{[1,0]}(3, 2, 2)m_c^2m_s^2 \\
& +6\hat{I}_1(1, 1, 4)m_c^2m_s^2 + 3\hat{I}_0(4, 1, 1)m_c^2m_s^2 - 6\hat{I}_2(1, 1, 4)m_c^2m_s^2 - 6\hat{I}_0(1, 1, 4)m_c^2m_s^2 \\
& -\hat{I}_2^{[1,0]}(3, 2, 2)m_c^2m_s^2 - \hat{I}_2^{[1,0]}(3, 2, 2)m_cm_s^3 + 2\hat{I}_2(3, 1, 2)m_cm_s^3 - \hat{I}_1^{[1,0]}(3, 2, 2)m_cm_s^3 \\
& -12\hat{I}_0(1, 1, 4)m_cm_s^3 - 2\hat{I}_1(3, 1, 2)m_cm_s^3 - 2\hat{I}_0^{[1,0]}(3, 2, 2)m_cm_s^3 + 8\hat{I}_0(3, 1, 2)m_cm_s^3 \\
& -6\hat{I}_0(1, 1, 4)m_s^4 + 3\hat{I}_0(3, 1, 2)m_s^4 + 3\hat{I}_0(2, 2, 1)m_c^2 - 3\hat{I}_1^{[0,1]}(3, 1, 2)m_c^2 \\
& -2\hat{I}_1(1, 2, 2)m_c^2 - 3\hat{I}_0^{[0,1]}(4, 1, 1)m_c^2 + \hat{I}_0^{[0,1]}(3, 2, 1)m_c^2 - 3\hat{I}_2^{[0,1]}(3, 1, 2)m_c^2 \\
& -\hat{I}_0^{[0,2]}(3, 2, 2)m_c^2 + 5\hat{I}_1^{[1,1]}(3, 2, 2)m_c^2 + 5\hat{I}_2^{[1,1]}(3, 2, 2)m_c^2 + 2\hat{I}_2(1, 2, 2)m_c^2 \\
& +2\hat{I}_1(1, 2, 2)m_cm_s - 3\hat{I}_2^{[1,0]}(3, 2, 1)m_cm_s - 2\hat{I}_2(1, 2, 2)m_cm_s - \hat{I}_2(2, 2, 1)m_cm_s \\
& +\hat{I}_1^{[1,0]}(3, 2, 2)m_cm_s + 2\hat{I}_0^{[1,1]}(3, 2, 2)m_cm_s + 6\hat{I}_0(3, 1, 1)m_cm_s + \hat{I}_1(2, 2, 1)m_cm_s \\
& -2\hat{I}_1^{[0,1]}(2, 1, 3)m_cm_s - 2\hat{I}_2^{[0,1]}(2, 1, 3)m_cm_s + \hat{I}_2^{[1,1]}(3, 2, 2)m_cm_s - 3\hat{I}_1^{[1,0]}(3, 2, 1)m_cm_s \\
& -\hat{I}_1^{[1,0]}(2, 2, 2)m_s^2 - 3\hat{I}_0^{[1,0]}(3, 2, 1)m_s^2 + 6\hat{I}_0^{[0,1]}(1, 1, 4)m_s^2 + 4\hat{I}_0(3, 1, 1)m_s^2 \\
& -5\hat{I}_0(2, 1, 2)m_s^2 - \hat{I}_2^{[1,0]}(2, 2, 2)m_s^2 - 2\hat{I}_0^{[1,0]}(2, 2, 2)m_s^2 - \hat{I}_2(2, 1, 2)m_s^2 + \hat{I}_1(2, 1, 2)m_s^2 \\
& -6\hat{I}_0^{[0,1]}(3, 1, 2)m_s^2 - \hat{I}_0^{[1,0]}(1, 2, 2) + 2\hat{I}_0^{[1,1]}(2, 2, 2) - 2\hat{I}_0^{[0,1]}(2, 2, 1) \\
& -2\hat{I}_2(2, 1, 1) + 3\hat{I}_0(2, 1, 1) - \hat{I}_0^{[0,1]}(1, 2, 2) - 6\hat{I}_0^{[0,1]}(1, 3, 1) + 2\hat{I}_1(2, 1, 1) \\
& +2\hat{I}_1(1, 2, 1) + 2\hat{I}_1(1, 1, 2) - \hat{I}_0(1, 2, 1) - \hat{I}_0^{[1,0]}(2, 2, 1) - 2\hat{I}_2(1, 2, 1) \\
& +\hat{I}_0^{[0,2]}(2, 2, 2) - 2\hat{I}_2(1, 1, 2) - 3\hat{I}_0^{[0,1]}(2, 1, 2),
\end{aligned}$$

$$\begin{aligned}
C_{D_s DK^*}^{K*} = & -\hat{I}_0(3, 2, 2)m_c^6 - \hat{I}_1(3, 2, 2)m_c^5m_s + \hat{I}_2(3, 2, 2)m_c^5m_s + 2\hat{I}_0(3, 2, 2)m_c^5m_s \\
& -\hat{I}_1(3, 2, 2)m_c^4m_s^2 + \hat{I}_2(3, 2, 2)m_c^4m_s^2 - 2\hat{I}_0(3, 2, 2)m_c^3m_s^3 - \hat{I}_2(3, 1, 2)m_c^4 \\
& -\hat{I}_0^{[0,1]}(3, 2, 2)m_c^4 + 3\hat{I}_1(2, 2, 2)m_c^4 - 3\hat{I}_2(2, 2, 2)m_c^4 + \hat{I}_1(3, 1, 2)m_c^4 \\
& -3\hat{I}_0(3, 2, 1)m_c^4 + 2\hat{I}_2(2, 2, 2)m_c^3m_s + 2\hat{I}_0(3, 2, 1)m_c^3m_s + 2\hat{I}_0^{[1,0]}(3, 2, 2)m_c^3m_s \\
& -2\hat{I}_1(2, 2, 2)m_c^3m_s + \hat{I}_2^{[1,0]}(3, 2, 2)m_c^3m_s + 6\hat{I}_0(4, 1, 1)m_c^3m_s + 4\hat{I}_0(2, 2, 2)m_c^3m_s \\
& +\hat{I}_2(3, 2, 1)m_c^3m_s - \hat{I}_1(3, 2, 1)m_c^3m_s + \hat{I}_0(3, 1, 2)m_c^3m_s + \hat{I}_1^{[1,0]}(3, 2, 2)m_c^3m_s \\
& -2\hat{I}_0(2, 2, 2)m_c^2m_s^2 + 2\hat{I}_1(3, 1, 2)m_c^2m_s^2 - 3\hat{I}_0(4, 1, 1)m_c^2m_s^2 - 2\hat{I}_2(3, 1, 2)m_c^2m_s^2
\end{aligned}$$

$$\begin{aligned}
& -2\hat{I}_0^{[0,1]}(3, 2, 2)m_c^2m_s^2 + 5\hat{I}_0(3, 1, 2)m_cm_s^3 + 2\hat{I}_2(2, 1, 3)m_cm_s^3 + 2\hat{I}_2(3, 1, 2)m_cm_s^3 \\
& -2\hat{I}_1(2, 1, 3)m_cm_s^3 - 2\hat{I}_1(3, 1, 2)m_cm_s^3 - \hat{I}_0(2, 2, 2)m_s^4 + \hat{I}_0^{[1,0]}(3, 2, 2)m_s^4 \\
& -5\hat{I}_1^{[1,1]}(3, 2, 2)m_c^2 + 2\hat{I}_0^{[0,1]}(3, 1, 2)m_c^2 - 5\hat{I}_2^{[1,1]}(3, 2, 2)m_c^2 + 3\hat{I}_2^{[1,0]}(3, 2, 1)m_c^2 \\
& +2\hat{I}_0^{[0,1]}(2, 2, 2)m_c^2 + 3\hat{I}_1(1, 3, 1)m_c^2 - 2\hat{I}_0(1, 2, 2)m_c^2 + 3\hat{I}_1^{[1,0]}(3, 2, 1)m_c^2 \\
& -3\hat{I}_2(1, 3, 1)m_c^2 - 12\hat{I}_0(1, 1, 3)m_cm_s + \hat{I}_2^{[1,1]}(3, 2, 2)m_cm_s - 3\hat{I}_1^{[1,0]}(3, 2, 1)m_cm_s \\
& -9\hat{I}_0(2, 1, 2)m_cm_s + 3\hat{I}_1(1, 3, 1)m_cm_s - 2\hat{I}_1^{[0,1]}(2, 1, 3)m_cm_s - \hat{I}_2(2, 2, 1)m_cm_s \\
& -3\hat{I}_2^{[1,0]}(3, 2, 1)m_cm_s + \hat{I}_1(2, 2, 1)m_cm_s + 4\hat{I}_1(2, 1, 2)m_cm_s - 2\hat{I}_2^{[0,1]}(2, 1, 3)m_cm_s \\
& -4\hat{I}_2(2, 1, 2)m_cm_s - 3\hat{I}_2(1, 3, 1)m_cm_s + \hat{I}_1^{[1,1]}(3, 2, 2)m_cm_s - 2\hat{I}_0(3, 1, 1)m_s^2 \\
& +\hat{I}_2(2, 1, 2)m_s^2 - \hat{I}_1(2, 1, 2)m_s^2 + 2\hat{I}_0^{[1,0]}(2, 2, 2)m_s^2 + 3\hat{I}_0(1, 1, 3)m_s^2 \\
& +\hat{I}_2^{[1,0]}(2, 2, 2)m_s^2 - 2\hat{I}_0(2, 2, 1)m_s^2 + \hat{I}_1^{[1,0]}(2, 2, 2)m_s^2 + 2\hat{I}_0^{[0,1]}(2, 2, 2)m_s^2 \\
& +3\hat{I}_0(2, 1, 2)m_s^2 - 2\hat{I}_0^{[1,1]}(2, 2, 2) - 2\hat{I}_0^{[0,2]}(3, 1, 2) - 2\hat{I}_1(1, 2, 1) + 2\hat{I}_0^{[0,1]}(2, 2, 1) \\
& +\hat{I}_0^{[0,1]}(1, 2, 2) + 2\hat{I}_2(1, 2, 1) - 3\hat{I}_0^{[1,1]}(3, 2, 1) - 3\hat{I}_0^{[0,1]}(1, 1, 3) + \hat{I}_0(2, 1, 1) \\
& +2\hat{I}_2(1, 1, 2) + \hat{I}_0^{[0,1]}(2, 1, 2) - 2\hat{I}_1(1, 1, 2),
\end{aligned}$$

$$\begin{aligned}
C_{D_s^* D^* K^*}^{K*} = & \hat{I}_0(3, 2, 2)m_c^6 - \hat{I}_0(3, 2, 2)m_c^5m_s + \hat{I}_0(3, 2, 2)m_c^3m_s^3 + \hat{I}_1^{[1,0]}(3, 2, 2)m_c^4 \\
& +3\hat{I}_0(4, 1, 1)m_c^4 - \hat{I}_2^{[1,0]}(3, 2, 2)m_c^4 + 2\hat{I}_6(3, 2, 2)m_c^4 + 2\hat{I}_2(2, 1, 3)m_c^3m_s \\
& +2\hat{I}_0(2, 1, 3)m_c^3m_s - 2\hat{I}_1(2, 2, 2)m_c^3m_s - 2\hat{I}_1(2, 1, 3)m_c^3m_s - \hat{I}_1(3, 2, 1)m_c^3m_s \\
& +\hat{I}_2(3, 2, 1)m_c^3m_s - 3\hat{I}_0(4, 1, 1)m_c^3m_s + 2\hat{I}_2(2, 2, 2)m_c^3m_s + \hat{I}_0(2, 2, 2)m_c^2m_s^2 \\
& -2\hat{I}_6(3, 2, 2)m_c^2m_s^2 + \hat{I}_0^{[0,1]}(3, 2, 2)m_c^2m_s^2 + \hat{I}_2^{[1,0]}(3, 2, 2)m_c^2m_s^2 - \hat{I}_1^{[1,0]}(3, 2, 2)m_c^2m_s^2 \\
& +2\hat{I}_2(3, 1, 2)m_cm_s^3 - \hat{I}_2^{[1,0]}(3, 2, 2)m_cm_s^3 - 2\hat{I}_1(3, 1, 2)m_cm_s^3 + 6\hat{I}_0(1, 1, 4)m_cm_s^3 \\
& +\hat{I}_1^{[1,0]}(3, 2, 2)m_cm_s^3 + \hat{I}_0(3, 1, 2)m_s^4 + 2\hat{I}_6^{[1,0]}(3, 2, 2)m_c^2 + \hat{I}_0(2, 1, 2)m_c^2 \\
& +8\hat{I}_8(3, 2, 1)m_c^2 + 4\hat{I}_8^{[0,1]}(3, 2, 2)m_c^2 + 2\hat{I}_6^{[0,1]}(3, 2, 2)m_c^2 - 4\hat{I}_6(3, 2, 1)m_c^2 \\
& -4\hat{I}_7^{[0,1]}(3, 2, 2)m_c^2 + 3\hat{I}_1^{[1,0]}(4, 1, 1)m_c^2 + 6\hat{I}_6(4, 1, 1)m_c^2 + 6\hat{I}_6(3, 1, 2)m_c^2 \\
& -8\hat{I}_7(3, 2, 1)m_c^2 - 12\hat{I}_7(4, 1, 1)m_c^2 + 2\hat{I}_0(2, 2, 1)m_c^2 + 12\hat{I}_8(4, 1, 1)m_c^2 \\
& -3\hat{I}_2^{[1,0]}(4, 1, 1)m_c^2 + 8\hat{I}_6(2, 1, 3)m_cm_s + 4\hat{I}_6(2, 2, 2)m_cm_s - \hat{I}_0^{[1,1]}(3, 2, 2)m_cm_s \\
& -3\hat{I}_1(1, 3, 1)m_cm_s - 2\hat{I}_1^{[0,1]}(3, 1, 2)m_cm_s + 3\hat{I}_1^{[1,0]}(3, 2, 1)m_cm_s - 2\hat{I}_6(3, 2, 1)m_cm_s \\
& +3\hat{I}_2(1, 3, 1)m_cm_s + 2\hat{I}_2^{[0,1]}(3, 1, 2)m_cm_s - \hat{I}_0(2, 1, 2)m_cm_s - 2\hat{I}_6(3, 1, 2)m_cm_s \\
& -3\hat{I}_2^{[1,0]}(3, 2, 1)m_cm_s - 2\hat{I}_6^{[1,0]}(3, 2, 2)m_s^2 - 12\hat{I}_6(1, 1, 4)m_s^2 - \hat{I}_0(2, 1, 2)m_s^2
\end{aligned}$$

$$\begin{aligned}
& -3\hat{I}_0(1, 1, 3)m_s^2 - 2\hat{I}_0^{[0,1]}(3, 1, 2)m_s^2 + 3\hat{I}_0^{[1,0]}(1, 1, 4)m_s^2 + \hat{I}_0^{[1,1]}(3, 2, 2)m_s^2 \\
& + 6\hat{I}_2^{[1,0]}(1, 1, 4)m_s^2 + \hat{I}_0(3, 1, 1)m_s^2 + 24\hat{I}_7(1, 1, 4)m_s^2 - 24\hat{I}_8(1, 1, 4)m_s^2 \\
& + 4\hat{I}_6(2, 2, 2)m_s^2 - 6\hat{I}_1^{[1,0]}(1, 1, 4)m_s^2 - 2\hat{I}_0^{[1,0]}(2, 2, 2)m_s^2 + 3\hat{I}_0^{[0,1]}(1, 1, 4)m_s^2 \\
& + 2\hat{I}_6(3, 1, 2)m_s^2 + 3\hat{I}_1^{[0,1]}(2, 1, 2) - 2\hat{I}_2(2, 1, 1) + 3\hat{I}_6(1, 3, 1) \\
& - 3\hat{I}_2^{[0,1]}(2, 1, 2) - 4S_1, 1(3, 2, 2) + 4\hat{I}_7^{[1,0]}(3, 2, 1) - 8\hat{I}_8(2, 2, 1) \\
& - 2\hat{I}_6^{[0,1]}(2, 2, 2) - 2\hat{I}_6^{[0,1]}(3, 1, 2) + 8\hat{I}_7(2, 2, 1) - 4\hat{I}_8^{[1,0]}(3, 2, 1) \\
& - 4\hat{I}_6^{[1,0]}(3, 2, 1) + 2\hat{I}_6^{[1,1]}(3, 2, 2) - 12\hat{I}_8(3, 1, 1) + 3\hat{I}_1^{[1,0]}(3, 1, 2) \\
& + 2\hat{I}_1(2, 1, 1) - 2\hat{I}_2^{[1,0]}(1, 2, 2) + 2\hat{I}_1^{[1,0]}(1, 2, 2) + 4\hat{I}_7^{[0,1]}(2, 2, 2) \\
& - 4\hat{I}_8^{[0,1]}(2, 2, 2) + 4\hat{I}_6(2, 2, 1) - \hat{I}_0^{[0,1]}(3, 1, 1) - 3\hat{I}_2^{[1,1]}(3, 1, 2) \\
& + \hat{I}_2^{[2,0]}(2, 2, 2) - \hat{I}_1^{[2,0]}(2, 2, 2) + 12\hat{I}_7(3, 1, 1) - 8\hat{I}_6(2, 1, 2) \\
& + 4\hat{I}_8^{[1,1]}(3, 2, 2) + \hat{I}_0^{[2,0]}(3, 2, 1) - 2\hat{I}_0^{[1,0]}(1, 2, 2) - \hat{I}_0^{[0,1]}(2, 1, 2) \\
& - 3\hat{I}_0(2, 1, 1) - 4\hat{I}_6(3, 1, 1) - 2\hat{I}_6^{[1,0]}(2, 2, 2) + 4\hat{I}_7^{[1,0]}(2, 2, 2) \\
& - 4\hat{I}_8^{[1,0]}(2, 2, 2) + 2\hat{I}_0(1, 1, 2) - \hat{I}_0^{[1,0]}(3, 1, 1) - 6\hat{I}_7(1, 3, 1) \\
& + 6\hat{I}_8(1, 3, 1),
\end{aligned}$$

$$\begin{aligned}
C_{D_{s1}D_1K^*}^{K*} = & \hat{I}_0(3, 2, 2)m_c^6 + \hat{I}_2(3, 2, 2)m_c^3m_s^3 - \hat{I}_1(3, 2, 2)m_c^3m_s^3 - \hat{I}_0(3, 2, 2)m_c^3m_s^3 \\
& - 4\hat{I}_7(3, 2, 2)m_c^4 + 3\hat{I}_0(2, 2, 2)m_c^4 + 2\hat{I}_6(3, 2, 2)m_c^4 + 4\hat{I}_8(3, 2, 2)m_c^4 \\
& + 3\hat{I}_0(4, 1, 1)m_c^4 - 2\hat{I}_2(2, 2, 2)m_c^3m_s + 2\hat{I}_1(2, 2, 2)m_c^3m_s - 2\hat{I}_2(2, 1, 3)m_c^3m_s \\
& + 2\hat{I}_1(2, 1, 3)m_c^3m_s + \hat{I}_1(3, 2, 1)m_c^3m_s + \hat{I}_0^{[0,1]}(3, 2, 2)m_c^3m_s - \hat{I}_2(3, 2, 1)m_c^3m_s \\
& - \hat{I}_0(3, 1, 2)m_c^3m_s - 2\hat{I}_6(3, 2, 2)m_c^2m_s^2 - \hat{I}_1^{[1,0]}(3, 2, 2)m_c^2m_s^2 - 6\hat{I}_0(1, 1, 4)m_c^2m_s^2 \\
& + \hat{I}_2^{[1,0]}(3, 2, 2)m_c^2m_s^2 + 6\hat{I}_2(1, 1, 4)m_cm_s^3 - 6\hat{I}_1(1, 1, 4)m_cm_s^3 + \hat{I}_0(3, 1, 2)m_s^4 \\
& + \hat{I}_1(2, 1, 2)m_c^2 - \hat{I}_1(3, 1, 1)m_c^2 + 2\hat{I}_0(2, 2, 1)m_c^2 + \hat{I}_1^{[2,0]}(3, 2, 2)m_c^2 \\
& + 8\hat{I}_8(3, 2, 1)m_c^2 + 4\hat{I}_8^{[0,1]}(3, 2, 2)m_c^2 + 2\hat{I}_6^{[0,1]}(3, 2, 2)m_c^2 - 12\hat{I}_7(4, 1, 1)m_c^2 \\
& - 4\hat{I}_7^{[0,1]}(3, 2, 2)m_c^2 + 4\hat{I}_8^{[1,0]}(3, 2, 2)m_c^2 + \hat{I}_2(3, 1, 1)m_c^2 + 2\hat{I}_8^{[1,0]}(3, 2, 2)m_c^2 \\
& - \hat{I}_2^{[2,0]}(3, 2, 2)m_c^2 - 4\hat{I}_6(3, 2, 1)m_c^2 - 8\hat{I}_7(3, 2, 1)m_c^2 - \hat{I}_0^{[0,1]}(2, 2, 2)m_c^2 \\
& + \hat{I}_0(2, 1, 2)m_c^2 + 6\hat{I}_6(4, 1, 1)m_c^2 - \hat{I}_2(2, 1, 2)m_c^2 - 4\hat{I}_7^{[1,0]}(3, 2, 2)m_c^2 \\
& + 6\hat{I}_6(3, 1, 2)m_c^2 + 12\hat{I}_8(4, 1, 1)m_c^2 - 4\hat{I}_1(1, 1, 3)m_cm_s + 4\hat{I}_2(1, 1, 3)m_cm_s \\
& - 4\hat{I}_6(2, 2, 2)m_cm_s - 3\hat{I}_2(1, 3, 1)m_cm_s + 2\hat{I}_0(1, 2, 2)m_cm_s + 2\hat{I}_6(3, 1, 2)m_cm_s
\end{aligned}$$

$$\begin{aligned}
& +\hat{I}_0(2,1,2)m_cm_s - 2\hat{I}_2^{[0,1]}(3,1,2)m_cm_s - 8\hat{I}_6(2,1,3)m_cm_s + 2\hat{I}_0^{[1,0]}(2,1,3)m_cm_s \\
& +2\hat{I}_6(3,2,1)m_cm_s + \hat{I}_2(2,1,2)m_cm_s + 2\hat{I}_1^{[0,1]}(3,1,2)m_cm_s - \hat{I}_1(2,1,2)m_cm_s \\
& +3\hat{I}_1(1,3,1)m_cm_s + 3\hat{I}_0(2,2,1)m_cm_s - 3\hat{I}_0(1,1,3)m_s^2 + 4\hat{I}_6(2,2,2)m_s^2 \\
& -6\hat{I}_1^{[1,0]}(1,1,4)m_s^2 + 3\hat{I}_0^{[0,1]}(1,1,4)m_s^2 - 12\hat{I}_6(1,1,4)m_s^2 - 2\hat{I}_0^{[0,1]}(3,1,2)m_s^2 \\
& -\hat{I}_0(2,1,2)m_s^2 + 6\hat{I}_2^{[1,0]}(1,1,4)m_s^2 + 2\hat{I}_6(3,1,2)m_s^2 - 2\hat{I}_6^{[1,0]}(3,2,2)m_s^2 \\
& -2\hat{I}_2^{[1,0]}(1,2,2) + \hat{I}_2^{[1,0]}(3,1,1) - 4\hat{I}_6^{[1,0]}(3,2,1) - \hat{I}_0^{[1,0]}(3,1,1) \\
& -8\hat{I}_8(2,2,1) - \hat{I}_0^{[0,1]}(2,1,2) - 12\hat{I}_8(3,1,1) - 2\hat{I}_6^{[1,0]}(2,2,2) + 12\hat{I}_7(3,1,1) \\
& -8\hat{I}_8(2,1,2) - 8\hat{I}_6(2,1,2) - \hat{I}_1^{[1,0]}(3,1,1) + 8\hat{I}_7(2,1,2) + 4\hat{I}_7^{[1,0]}(3,2,1) \\
& -4\hat{I}_8^{[1,0]}(3,2,1) + 4\hat{I}_7^{[0,1]}(3,1,2) - 4\hat{I}_8^{[0,1]}(3,1,2) + 2\hat{I}_1^{[0,1]}(1,2,2) \\
& +2\hat{I}_1(2,1,1) - 4\hat{I}_7^{[1,1]}(3,2,2) + 4\hat{I}_8^{[1,1]}(3,2,2) + 3\hat{I}_6(1,3,1) - 2\hat{I}_6^{[0,1]}(3,1,2) \\
& -2\hat{I}_6^{[0,1]}(2,2,2) - 4\hat{I}_6(3,1,1) + \hat{I}_0^{[0,2]}(3,1,2) - \hat{I}_0^{[0,1]}(3,1,1) \\
& -3\hat{I}_0(2,1,1) + 4\hat{I}_6(2,2,1) + 2\hat{I}_6^{[1,1]}(3,2,2) - 2\hat{I}_2(2,1,1) \\
& +8\hat{I}_7(2,2,1) - 2\hat{I}_0^{[0,1]}(2,2,1),
\end{aligned}$$

$$\begin{aligned}
C_{D_s D^* K^*}^{K^*} = & 2\hat{I}_2(3,2,2)m_c^5 + 2\hat{I}_0(3,2,2)m_c^5 + 2\hat{I}_1(3,2,2)m_c^5 - 2\hat{I}_2(3,2,2)m_c^4 m_s \\
& -2\hat{I}_0(3,2,2)m_c^3 m_s^2 - 2\hat{I}_2(3,2,2)m_c^3 m_s^2 - 2\hat{I}_1(3,2,2)m_c^3 m_s^2 + 2\hat{I}_2(3,2,2)m_c^2 m_s^3 \\
& +4\hat{I}_2(2,2,2)m_c^3 + 2\hat{I}_0^{[0,1]}(3,2,2)m_c^3 + 6\hat{I}_1(4,1,1)m_c^3 + 2\hat{I}_2^{[1,0]}(3,2,2)m_c^3 \\
& +6\hat{I}_0(4,1,1)m_c^3 + 4\hat{I}_0(2,2,2)m_c^3 + 2\hat{I}_0(3,2,1)m_c^3 + 2\hat{I}_0^{[1,0]}(3,2,2)m_c^3 \\
& +2\hat{I}_1^{[0,1]}(3,2,2)m_c^3 + 2\hat{I}_2^{[0,1]}(3,2,2)m_c^3 - 2\hat{I}_1(3,1,2)m_c^3 + 2\hat{I}_1(3,2,1)m_c^3 \\
& +2\hat{I}_1^{[1,0]}(3,2,2)m_c^3 + 4\hat{I}_1(2,2,2)m_c^3 + 6\hat{I}_2(4,1,1)m_c^3 - 4\hat{I}_2(2,2,2)m_c^2 m_s \\
& -2\hat{I}_2^{[1,0]}(3,2,2)m_c^2 m_s + 8\hat{I}_0(2,1,3)m_c^2 m_s + 2\hat{I}_1(3,1,2)m_c^2 m_s - 2\hat{I}_0(3,1,2)m_c^2 m_s \\
& +4\hat{I}_2(2,1,3)m_c^2 m_s - 2\hat{I}_2(3,1,2)m_c^2 m_s - 6\hat{I}_2(4,1,1)m_c^2 m_s - 2\hat{I}_2^{[0,1]}(3,2,2)m_c^2 m_s \\
& +4\hat{I}_1(3,1,2)m_c m_s^2 + 4\hat{I}_0(3,1,2)m_c m_s^2 - 2\hat{I}_1^{[1,0]}(3,2,2)m_c m_s^2 + 12\hat{I}_1(1,1,4)m_c m_s^2 \\
& -2\hat{I}_2^{[1,0]}(3,2,2)m_c m_s^2 + 12\hat{I}_2(1,1,4)m_c m_s^2 - 2\hat{I}_0^{[1,0]}(3,2,2)m_c m_s^2 + 6\hat{I}_2(3,1,2)m_c m_s^2 \\
& +12\hat{I}_0(1,1,4)m_c m_s^2 - 4\hat{I}_2(3,1,2)m_s^3 + 2\hat{I}_2^{[1,0]}(3,2,2)m_s^3 - 12\hat{I}_2(1,1,4)m_s^3 \\
& -4\hat{I}_2(2,1,3)m_s^3 - 2\hat{I}_2(2,2,2)m_s^3 + 2\hat{I}_2(3,1,1)m_c - 4\hat{I}_2^{[1,0]}(3,2,1)m_c \\
& -4\hat{I}_1^{[0,1]}(3,1,2)m_c + 4\hat{I}_0(1,2,2)m_c - 2\hat{I}_1^{[1,0]}(3,1,2)m_c + 4\hat{I}_2(1,2,2)m_c \\
& +8\hat{I}_0(2,1,2)m_c + 8\hat{I}_2(2,1,2)m_c + 2\hat{I}_0(3,1,1)m_c + 4\hat{I}_1(1,2,2)m_c
\end{aligned}$$

$$\begin{aligned}
& -6\hat{I}_1^{[1,0]}(3,2,1)m_c + 2\hat{I}_0(2,2,1)m_c - 4\hat{I}_0^{[0,1]}(3,1,2)m_c - 4\hat{I}_2^{[0,1]}(3,1,2)m_c \\
& + 2\hat{I}_2^{[1,1]}(3,2,2)m_c + 2\hat{I}_1^{[1,1]}(3,2,2)m_c + 2\hat{I}_1(2,2,1)m_c + 6\hat{I}_1(2,1,2)m_c \\
& + 2\hat{I}_0^{[1,1]}(3,2,2)m_c - 6\hat{I}_0^{[1,0]}(3,2,1)m_c - 2\hat{I}_1(3,1,1)m_c + 8\hat{I}_1^{[1,0]}(2,1,3)m_s \\
& + 4\hat{I}_2^{[1,0]}(3,2,1)m_s + 12\hat{I}_1(1,1,3)m_s - 4\hat{I}_2(1,2,2)m_s + 2\hat{I}_2^{[0,1]}(2,2,2)m_s \\
& - 2\hat{I}_2(2,2,1)m_s + 4\hat{I}_2^{[0,1]}(3,1,2)m_s - 2\hat{I}_2^{[1,1]}(3,2,2)m_s + 4\hat{I}_1(2,1,2)m_s \\
& + 4\hat{I}_2(1,1,3)m_s + 4\hat{I}_0(2,1,2)m_s - 10\hat{I}_2(2,1,2)m_s + 2\hat{I}_2^{[1,0]}(2,2,2)m_s \\
& + 4\hat{I}_2^{[0,1]}(2,1,3)m_s + 20\hat{I}_0(1,1,3)m_s,
\end{aligned}$$

$$\begin{aligned}
C_{D_{s_0}^* D_1 K^*}^{K^*} = & 2\hat{I}_1(3,2,2)m_c^5 + 2\hat{I}_2(3,2,2)m_c^5 + 2\hat{I}_0(3,2,2)m_c^5 + 2\hat{I}_2(3,2,2)m_c^4 m_s \\
& - 2\hat{I}_2(3,2,2)m_c^3 m_s^2 - 2\hat{I}_1(3,2,2)m_c^3 m_s^2 - 2\hat{I}_0(3,2,2)m_c^3 m_s^2 - 2\hat{I}_2(3,2,2)m_c^2 m_s^3 \\
& + 2\hat{I}_1^{[1,0]}(3,2,2)m_c^3 - 2\hat{I}_2(3,2,1)m_c^3 + 2\hat{I}_2^{[0,1]}(3,2,2)m_c^3 + 2\hat{I}_0^{[0,1]}(3,2,2)m_c^3 \\
& + 4\hat{I}_1(2,2,2)m_c^3 + 2\hat{I}_0^{[1,0]}(3,2,2)m_c^3 + 4\hat{I}_0(2,2,2)m_c^3 + 6\hat{I}_2(4,1,1)m_c^3 \\
& + 2\hat{I}_2^{[1,0]}(3,2,2)m_c^3 + 4\hat{I}_2(2,2,2)m_c^3 + 2\hat{I}_2(3,1,2)m_c^3 + 2\hat{I}_0(3,1,2)m_c^3 \\
& + 6\hat{I}_1(4,1,1)m_c^3 + 2(\hat{I}_0^{[0,1]}(3,2,2)m_c^3 + 6\hat{I}_0(4,1,1)m_c^3 + 2\hat{I}_2^{[0,1]}(3,2,2)m_c^2 m_s \\
& + 6\hat{I}_2(4,1,1)m_c^2 m_s - 2\hat{I}_1(2,2,2)m_c^2 m_s + 2\hat{I}_0(3,1,2)m_c^2 m_s - 2\hat{I}_1(3,1,2)m_c^2 m_s \\
& + 8\hat{I}_0(2,1,3)m_c^2 m_s + 2\hat{I}_2(3,1,2)m_c^2 m_s + 8\hat{I}_1(2,1,3)m_c^2 m_s + 2\hat{I}_2(2,2,2)m_c^2 m_s \\
& + 2\hat{I}_2^{[1,0]}(3,2,2)m_c^2 m_s + 4\hat{I}_2(2,1,3)m_c^2 m_s - 2\hat{I}_0(2,2,2)m_c^2 m_s - 2\hat{I}_1^{[1,0]}(3,2,2)m_c m_s^2 \\
& + 12\hat{I}_1(1,1,4)m_c m_s^2 + 6\hat{I}_0(3,1,2)m_c m_s^2 + 4\hat{I}_1(3,1,2)m_c m_s^2 + 12\hat{I}_2(1,1,4)m_c m_s^2 \\
& + 2\hat{I}_2(3,2,1)m_c m_s^2 - 2\hat{I}_0^{[1,0]}(3,2,2)m_c m_s^2 - 2\hat{I}_2^{[1,0]}(3,2,2)m_c m_s^2 + 12\hat{I}_0(1,1,4)m_c m_s^2 \\
& + 4\hat{I}_2(3,1,2)m_c m_s^2 - 2\hat{I}_2^{[1,0]}(3,2,2)m_s^3 + 4\hat{I}_2(3,1,2)m_s^3 + 12\hat{I}_2(1,1,4)m_s^3 \\
& + 4\hat{I}_2(2,2,2)m_s^3 - 4\hat{I}_2(2,1,3)m_s^3 - 6\hat{I}_2^{[0,1]}(3,1,2)m_c - 2\hat{I}_2(2,2,1)m_c \\
& + 4\hat{I}_0(1,2,2)m_c - 4\hat{I}_1^{[1,0]}(3,2,1)m_c + 4\hat{I}_2(1,2,2)m_c + 10\hat{I}_0(2,1,2)m_c \\
& + 2\hat{I}_1^{[1,1]}(3,2,2)m_c + 4\hat{I}_1(1,2,2)m_c + 8\hat{I}_1(2,1,2)m_c + 10\hat{I}_2(2,1,2)m_c \\
& - 4\hat{I}_2^{[1,0]}(3,2,1)m_c - 4\hat{I}_1^{[0,1]}(3,1,2)m_c - 6\hat{I}_0^{[0,1]}(3,1,2)m_c + 2\hat{I}_0(3,1,1)m_c \\
& + 2\hat{I}_0^{[1,1]}(3,2,2)m_c - 2\hat{I}_2^{[0,1]}(3,2,1)m_c - 4\hat{I}_0^{[1,0]}(3,2,1)m_c + 2\hat{I}_2^{[1,1]}(3,2,2)m_c \\
& + 2\hat{I}_2(1,2,2)m_s - 4\hat{I}_2^{[1,0]}(3,2,1)m_s - 2\hat{I}_1(1,2,2)m_s + 2\hat{I}_0^{[1,0]}(2,2,2)m_s \\
& + 6\hat{I}_1(1,3,1)m_s + 4\hat{I}_2(1,1,3)m_s - 2\hat{I}_2^{[1,0]}(2,2,2)m_s - 4\hat{I}_1(1,1,3)m_s \\
& - 2\hat{I}_1(2,1,2)m_s - 4\hat{I}_2^{[0,1]}(2,2,2)m_s - 6\hat{I}_0(2,1,2)m_s - 2\hat{I}_0(1,2,2)m_s
\end{aligned}$$

$$\begin{aligned}
& +4\hat{I}_2^{[0,1]}(2, 1, 3)m_s - 4\hat{I}_0(1, 1, 3)m_s + 2\hat{I}_1^{[1,0]}(2, 2, 2)m_s - 4\hat{I}_2^{[0,1]}(3, 1, 2)m_s \\
& +2\hat{I}_2^{[1,1]}(3, 2, 2)m_s + 2\hat{I}_2(2, 1, 2)m_s + 4\hat{I}_2(2, 2, 1)m_s,
\end{aligned}$$

where

$$\begin{aligned}
\hat{I}_\mu^{[\alpha,\beta]}(a, b, c) &= [M_1^2]^\alpha [M_2^2]^\beta \frac{d^\alpha}{d(M_1^2)^\alpha} \frac{d^\beta}{d(M_2^2)^\beta} [M_1^2]^\alpha [M_2^2]^\beta \hat{I}_\mu(a, b, c), \\
\hat{I}_k(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{1-a-b+k} (M_2^2)^{4-a-c-k} U_0(a+b+c-5, 1-c-b), \\
\hat{I}_m(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{-a-b-1+m} (M_2^2)^{7-a-c-m} U_0(a+b+c-5, 1-c-b), \\
\hat{I}_6(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{32\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} U_0(a+b+c-6, 2-c-b), \\
\hat{I}_n(a, b, c) &= i \frac{(-1)^{a+b+c}}{32\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{-4-a-b+n} (M_2^2)^{11-a-c-n} U_0(a+b+c-7, 2-c-b),
\end{aligned}$$

where $k = 1, 2$, $m = 3, 4, 5$ and $n = 7, 8$. We can define the function $U_0(a, b)$ as:

$$U_0(a, b) = \int_0^\infty dy (y + M_1^2 + M_2^2)^a y^b \exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right],$$

where

$$\begin{aligned}
B_{-1} &= \frac{1}{M_2^2 M_1^2} (m_s^2 (M_1^2 + M_2^2)^2 - M_2^2 M_1^2 Q^2), \\
B_0 &= \frac{1}{M_1^2 M_2^2} (m_s^2 + m_c^2) (M_1^2 + M_2^2), \\
B_1 &= \frac{m_c^2}{M_1^2 M_2^2}.
\end{aligned}$$

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